

Calculators and mobile phones are not allowed.

Answer all of the following questions.

1. Use differentials to approximate the change in the area of a square if the length of each side changes from 9 cm. to 9.01 cm. (4 points)
2. Find the points on the graph of the equation

$$x^2 - 2xy + 2y^2 = 4$$

at which the tangent line is horizontal.

(4 points)

3. The area of a rectangle is increasing at a rate of $20 \text{ cm}^2/\text{sec}$, while its width is decreasing at a rate of $2 \text{ cm}/\text{sec}$. Find the rate of change of its length when the rectangle is a square of area 100 cm^2 . (4 points)

4. State the mean value theorem and use it to prove that :
if $|f'(x)| < M$ for all x in (a, b) , and if x_1 and x_2 are any two points in (a, b) with $x_1 < x_2$, then

$$|f(x_2) - f(x_1)| \leq M|x_2 - x_1|.$$

(4 points)

5. Let $f(x) = \frac{1-2x}{x^2}$.

- (a) Find the intervals on which f is increasing or decreasing, and find the local extrema, if any.
- (b) Find the intervals on which the graph of f is concave upward or downward, and find the points of inflection, if any.
- (c) Find the vertical and horizontal asymptotes for the graph of f , if any.
- (d) Sketch the graph of f .

(9 points)

(Good Luck)